

---

# **pyModelChecking Documentation**

***Release 0.2.0***

**Alberto Casagrande**

**Apr 19, 2019**



---

## Contents

---

<b>1</b>	<b>Basic Notions</b>	<b>3</b>
1.1	Reactive Systems . . . . .	3
1.2	Temporal Logics . . . . .	4
1.3	Model Checking . . . . .	7
1.4	Symbolic Representation . . . . .	8
<b>2</b>	<b>Using <i>pyModelChecking</i></b>	<b>9</b>
2.1	Modelling Reactive Systems . . . . .	9
2.2	Encoding Formulas and Model Checking . . . . .	11
<b>3</b>	<b>API</b>	<b>15</b>
3.1	Models API . . . . .	15
3.2	Logics and Model Checking API . . . . .	15
<b>4</b>	<b>Indices and tables</b>	<b>17</b>
	<b>Bibliography</b>	<b>19</b>



*pyModelChecking* is a simple Python model checking package. Currently, it is able to represent *Kripke structures*, *CTL*, *LTL*, and *CTL\** formulas and it provides *model checking* methods for LTL, CTL, and CTL\*. In future, it will hopefully support symbolic model checking.



### 1.1 Reactive Systems

**Reactive systems** are systems that interact with their environment and evolve over an infinite time horizon. This chapter presents a natural model for them: Kripke structure.

#### 1.1.1 Directed Graphs

A **directed graph**, or **graph**, is pair  $(V, E)$  where:

- $V$  is a finite set of *nodes*
- $E \subseteq V \times V$  is a set of *edges*

If  $(s, d) \in E$ , then  $s$  and  $d$  are the *source* and the *destination* of  $(s, d)$ , respectively. The edge  $(s, d) \in E$  is said to *go from*  $s$  *to*  $d$ . If  $e \in E$  goes either from  $s$  to  $d$  or from  $d$  to  $s$ , then  $e$  is an edge **between**  $d$  and  $s$ . By extension, an edge  $e \in E$  goes from  $V_1 \subseteq S$  to  $V_2 \subseteq S$  if there exists a pair of nodes  $(v_1, v_2) \in V_1 \times V_2$  such that  $(v_1, v_2) \in E$ . Analogously,  $e \in E$  is between  $V_1$  and  $V_2$  if it is either from  $V_1$  to  $V_2$  or from  $V_2$  to  $V_1$ .

The **reversed graph** of a graph  $(V, E)$  is the graph  $(V, E')$  where  $E' = (d, s) | (s, d) \in E$ .

A **subgraph** of a graph  $(V, E)$  is a graph  $(V', E')$  such that  $V' \subseteq V$  and  $E' \subseteq E \cap (V' \times V')$ . A subgraph  $(V', E')$  of  $(V, E)$  is a **proper subgraph** if either  $V' \subsetneq V$  or  $E' \subsetneq E$ . A subgraph  $G$  of  $(V, E)$  **respects** a set of nodes  $V' \subseteq V$  if  $G = (V', E \cap (V' \times V'))$ .

A sequence, either finite or infinite,  $\pi = v_0 v_1 \dots$  is a **path** for the graph  $(V, E)$  if  $(v_i, v_{i+1}) \in E$  for all  $v_i$  and  $v_{i+1}$  in  $\pi$ . The *length of a path*  $\pi$ , denoted by  $|\pi|$ , is the size of the sequence.

It is easy to see that if  $\pi = v_0 \dots v_n$  and  $\pi' = w_0 \dots$  are two paths for  $(V, E)$  such that  $(v_n, w_0) \in E$ , then  $\pi \cdot \pi' = v_0 \dots v_n w_0 \dots$  is path for  $(V, E)$ .

Let  $\pi$ ,  $\pi'$ , and  $\pi''$  be three paths such that  $\pi = \pi' \cdot \pi''$ . Then,  $\pi'$  is a **prefix** of  $\pi$  and  $\pi''$  is a **suffix** of  $\pi$ . We write  $\pi_i$  to denote the suffix of  $\pi$  for which  $\pi = \pi' \cdot \pi_i$  and  $|\pi'| = i$  for some  $\pi'$ .

If  $v_0 v_1 \dots v_n$  is a prefix for some path  $\pi$  of a graph  $(V, E)$ , then we say that either  $\pi$  *starts* from  $v_0$  and **reaches**  $v_n$  or, equivalently,  $v_n$  is **reachable** from  $v_0$  in  $(V, E)$ .

Every *subgraph*  $(V', E')$  of  $G$  such that:

1.  $v$  is reachable from  $v'$  for all pairs  $v, v' \in V'$  and
2. is not proper subgraph of any subgraph of  $G$  that satisfies 1.

is a **strongly connected component** of  $G$ . It is easy to see that the sets of nodes of each strongly connected component of a graph  $(V, E)$  is a partition of  $V$ . A strongly connected component  $(V', E')$  is **trivial** if  $|V'| = 1$  and  $|E'| = 0$ .

## Directed Acyclic Graphs and Trees

A **directed acyclic graph** or **DAG** is a directed graph whose strongly connected components are all trivial.

A graph  $(V, E)$  is **disconnected** if there exists a  $V' \subseteq V$  such that there are no edges between  $V'$  and  $V \setminus V'$ . If a graph is not disconnected, then is **connected**.

A **directed tree** is a connected DAG  $(V, E)$  whose subgraphs of the form  $(V, E')$ , where  $E' \subsetneq E$ , are disconnected.

### 1.1.2 Kripke Structures

A **Kripke structure** is a *directed graph*, equipped with a set of initial nodes, such that every node is source of some edge and it is labeled by a set of *atomic propositions* [CGP00]. The nodes of Kripke structure are called *states*.

A Kripke structure is a tuple  $(S, S_0, R, L)$  such that:

- $S$  is a finite set of states
- $S_0 \subseteq S$  is a set of *initial states*
- $R \subseteq S \times S$  is a set of *transitions* such that for all  $s \in S$  there exists a  $(s, s') \in R$  for some  $s' \in S$
- $L : S \rightarrow AP$  maps each state into a set of atomic propositions

Sometime, the set of initial states is omitted. In such cases,  $S$  and  $S_0$  coincide.

A **computation** of a Kripke structure  $(S, S_0, R, L)$  is an infinite path of  $(S, R)$  that starts from some  $s \in S_0$ .

## 1.2 Temporal Logics

### 1.2.1 Computational Tree Logic\*

The **Computational Tree Language\*** or **CTL\*** is a the temporal logic that describes the properties of computation trees over Kripke structures ([CE81], [CES86]). Beside a set of atomic propositions and the standard logical operators  $\neg$ ,  $\wedge$ ,  $\vee$ , and  $\rightarrow$ , the alphabet of CTL\* contains the two path quantifiers **A** (“for all paths”) and **E** (“for some path”) and the five temporal operators **X** (“at the next step”), **G** (“globally”), **F** (“in the future”), **U** (“until”), and **R** (“release”).

#### Syntax

Any CTL\* formula is either a *state formula* (i.e., a formula that are evaluated in a single state) or a *path formula* (i.e., a formula whose truth value depend on an infinite path).

A CTL\* state formula is either:

- $\top$  or  $\perp$
- an atomic proposition



- $\neg\varphi_1, \varphi_1 \wedge \varphi_2, \varphi_1 \vee \varphi_2$ , or  $\varphi_1 \rightarrow \varphi_2$  where both  $\varphi_1$  and  $\varphi_2$  are CTL\* state formulas
- $\mathbf{A}\psi$  or  $\mathbf{E}\psi$  where  $\psi$  is a CTL\* path formula

A CTL\* path formula is either:

- a state formula
- $\neg\psi_1, \psi_1 \wedge \psi_2, \psi_1 \vee \psi_2$ , or  $\psi_1 \rightarrow \psi_2$  where both  $\psi_1$  and  $\psi_2$  are CTL\* path formulas
- $\mathbf{X}\psi_1, \mathbf{F}\psi_1, \mathbf{G}\psi_1, \psi_1 \mathbf{U}\psi_2$ , or  $\psi_1 \mathbf{R}\psi_2$  where both  $\psi_1$  and  $\psi_2$  are CTL\* path formulas

## Semantics

The semantics of CTL\* formulas are given with respect to a *Kripke structure*. If  $K$  is a Kripke structure,  $s$  one of its states, and  $\varphi$  a state formula, we write  $K, s \models \varphi$  (to be read “ $K$  and  $s$  satisfy  $\varphi$ ”) meaning that  $\varphi$  holds at state  $s$  in  $K$ . Analogously, If  $K$  is a Kripke structure,  $\pi$  one of its computations, and  $\psi$  a path formula, we write  $K, \pi \models \psi$  meaning that  $\psi$  holds along  $\pi$  in  $K$ .

Let  $K$  be the Kripke structure  $(S, S_0, R, L)$ ; the relation  $\models$  is defined recursively as follows:

- $K, s \models \top$  and  $K, s \not\models \perp$  for any state  $s \in S$
- if  $p \in AP$ , then  $K, s \models p \iff p \in L(s)$
- $K, s \models \neg\varphi \iff K, s \not\models \varphi$
- $K, s \models \varphi_1 \wedge \varphi_2 \iff K, s \models \varphi_1$  and  $K, s \models \varphi_2$
- $K, s \models \varphi_1 \vee \varphi_2 \iff K, s \models \varphi_1$  or  $K, s \models \varphi_2$
- $K, s \models \varphi_1 \rightarrow \varphi_2 \iff K, s \not\models \varphi_1$  or  $K, s \models \varphi_2$
- $K, s \models \mathbf{A}\varphi \iff K, \pi \models \varphi$  for any computation  $\pi$  of  $K$  that starts from  $s$
- $K, s \models \mathbf{E}\varphi \iff K, \pi \models \varphi$  for some computation  $\pi$  of  $K$  that starts from  $s$
- $K, \pi \models \psi \iff K, s \models \psi$ , where  $\pi$  is a computation of  $K$  that starts from  $s$
- $K, \pi \models \neg\psi \iff K, \pi \not\models \psi$
- $K, \pi \models \psi_1 \wedge \psi_2 \iff K, \pi \models \psi_1$  and  $K, \pi \models \psi_2$
- $K, \pi \models \psi_1 \vee \psi_2 \iff K, \pi \models \psi_1$  or  $K, \pi \models \psi_2$
- $K, \pi \models \psi_1 \rightarrow \psi_2 \iff K, \pi \not\models \psi_1$  or  $K, \pi \models \psi_2$
- $K, \pi \models \mathbf{X}\psi \iff K, \pi_1 \models \psi$
- $K, \pi \models \mathbf{F}\psi \iff K, \pi_i \models \psi$  for some  $i \in \mathbb{N}$
- $K, \pi \models \mathbf{G}\psi \iff K, \pi_i \models \psi$  for all  $i \in \mathbb{N}$
- $K, \pi \models \psi_1 \mathbf{U}\psi_2 \iff$  there exists an  $i \in \mathbb{N}$  such that  $K, \pi_i \models \psi_2$  and  $K, \pi_j \models \psi_1$  for all  $j \in [0, i - 1]$
- $K, \pi \models \psi_1 \mathbf{R}\psi_2 \iff$  for all  $i \in \mathbb{N}$ , if  $K, \pi_j \not\models \psi_1$  for all  $j \in [0, i - 1]$ , then  $K, \pi_i \models \psi_2$

Whenever  $K, \sigma \models \psi \iff K, \sigma \models \varphi$  for any  $\sigma$  and any  $K$ , we say that  $\psi$  and  $\varphi$  are **equivalent** and we write  $\varphi \equiv \psi$ .

Two set of formulas  $\mathcal{F}$  and any  $\mathbf{G}$  are **equivalent** if any formula  $\mathbf{G}$  has an equivalent formula in  $\mathcal{F}$  and vice versa.

## Restricted Syntax

It is easy to prove that  $\perp$ ,  $\mathbf{F}\psi$ ,  $\mathbf{G}\psi$ ,  $\varphi\mathbf{R}\psi$ ,  $\mathbf{A}\varphi$ ,  $\varphi \wedge \psi$ , and  $\varphi \rightarrow \psi$  are equivalent to  $\neg\top$ ,  $\top\mathbf{U}\psi$ ,  $\neg(\top\mathbf{U}\neg\psi)$ ,  $\neg(\neg\varphi\mathbf{U}\neg\psi)$ ,  $\neg\mathbf{E}\neg\varphi$ ,  $\neg(\varphi \vee \psi)$ , and  $\neg\varphi \vee \psi$ , respectively. Thus, the CTL\* language whose alphabet is restricted to  $\neg$ ,  $\vee$ ,  $\mathbf{X}$ ,  $\mathbf{U}$ ,  $\mathbf{A}$ ,  $\top$ , and atomic propositions is equivalent to the full CTL\* language (e.g., see [CGP00]).

### 1.2.2 Computational Tree Logic

The **Computational Tree Language** or **CTL** is a subset of *CTL\** ([BMP83], [CE81], [CE80]). In CTL, each occurrence of the two path quantifiers  $\mathbf{A}$  and  $\mathbf{E}$  should be coupled to one of the temporal operators  $\mathbf{X}$ ,  $\mathbf{G}$ ,  $\mathbf{F}$ ,  $\mathbf{U}$ , or  $\mathbf{U}$ .

#### Syntax

More formally, a CTL state formula is either:

- $\top$  or  $\perp$
- an atomic proposition
- $\neg\varphi_1$ ,  $\varphi_1 \wedge \varphi_2$ ,  $\varphi_1 \vee \varphi_2$ , or  $\varphi_1 \rightarrow \varphi_2$ , where both  $\varphi_1$  and  $\varphi_2$  are CTL state formulas
- $\mathbf{A}\psi$  or  $\mathbf{E}\psi$  where  $\varphi$  is a CTL path formula

A CTL path formula is either  $\mathbf{X}\varphi_1$ ,  $\mathbf{F}\varphi_1$ ,  $\mathbf{G}\varphi_1$ ,  $\varphi_1\mathbf{U}\varphi_2$ , or  $\varphi_1\mathbf{R}\varphi_2$  where both  $\varphi_1$  and  $\varphi_2$  are CTL state formulas.

#### Semantics

CTL has the same *semantics of CTL\**.

#### Restricted Syntax

Despite the apperent syntatic complexity of CTL, any possible property definable in it can be expressed by a CTL formula whose syntax is restricted to the use of  $\top$ ,  $\neg$ ,  $\vee$ , and  $\mathbf{E}$  coupled to either  $\mathbf{X}$ ,  $\mathbf{U}$ , or  $\mathbf{G}$  (e.g., see [CGP00]). As a matter of the facts, it is easy to prove that:

- $\perp \equiv \neg\top$
- $\varphi_1 \wedge \varphi_2 \equiv \neg(\neg\varphi_1 \vee \neg\varphi_2)$
- $\varphi_1 \rightarrow \varphi_2 \equiv \neg\varphi_1 \vee \varphi_2$
- $\mathbf{A}\mathbf{X}\varphi \equiv \neg\mathbf{E}\mathbf{X}(\neg\varphi)$
- $\mathbf{E}\mathbf{F}\varphi \equiv \mathbf{E}(\top\mathbf{U}\varphi)$
- $\mathbf{A}\mathbf{G}\varphi \equiv \neg\mathbf{E}(\top\mathbf{U}\neg\varphi)$
- $\mathbf{A}\mathbf{F}\varphi \equiv \neg\mathbf{E}\mathbf{G}(\neg\varphi)$
- $\mathbf{A}(\varphi_1\mathbf{U}\varphi_2) \equiv \neg(\mathbf{E}((\neg\varphi_2)\mathbf{U}\neg(\varphi_1 \vee \varphi_2)) \vee \mathbf{E}\mathbf{G}(\neg\varphi_2))$
- $\mathbf{A}(\varphi_1\mathbf{R}\varphi_2) \equiv \neg\mathbf{E}((\neg\varphi_1)\mathbf{U}(\neg\varphi_2))$
- $\mathbf{E}(\varphi_1\mathbf{R}\varphi_2) \equiv (\mathbf{E}(\varphi_2\mathbf{U}(\neg\varphi_1 \vee \neg\varphi_2)) \vee \mathbf{E}\mathbf{G}(\varphi_2))$

### 1.2.3 Linear Time Logic

The **Linear Time Logic** or **LTL** is a subset of of *CTL\** ([P77]).

#### Syntax

LTL formulas have the form  $A\rho$  where  $\rho$  is a LTL path formula and a LTL path formula is either:

- $\top$  or  $\perp$
- an atomic proposition
- $\neg\varphi_1$ ,  $\varphi_1 \wedge \varphi_2$ ,  $\varphi_1 \vee \varphi_2$ , or  $\varphi_1 \rightarrow \varphi_2$ , where both  $\varphi_1$  and  $\varphi_2$  are LTL path formulas
- $\mathbf{X}\varphi_1$ ,  $\mathbf{F}\varphi_1$ ,  $\mathbf{G}\varphi_1$ ,  $\varphi_1 \mathbf{U}\varphi_2$ , or  $\varphi_1 \mathbf{R}\varphi_2$  where both  $\varphi_1$  and  $\varphi_2$  are LTL path formulas.

#### Semantics

LTL has the same *semantics of CTL\**.

#### Restricted Syntax

It is easy to prove that:

- $\psi_1 \wedge \psi_2 \equiv \neg(\neg\psi_1 \vee \neg\psi_2)$
- $\psi_1 \rightarrow \psi_2 \equiv \neg\psi_1 \vee \psi_2$
- $\mathbf{F}\psi \equiv \top \mathbf{U}\psi$
- $\mathbf{G}\psi \equiv \neg(\top \mathbf{U}\neg\psi)$
- $\psi_1 \mathbf{R}\psi_2 \equiv \neg((\neg\psi_1) \mathbf{U}(\neg\psi_2))$

Hence, the LTL restricted language that allows exclusively the path formulas whose operators are  $\neg$ ,  $\vee$ ,  $\mathbf{X}$ , or  $\mathbf{U}$  is equivalent to the full LTL language (e.g., see [CGP00]).

## 1.3 Model Checking

Model checking is a technique to establish the set of states in Kripke structure that satisfy a given temporal formula. More formally, provided a Kripke structure  $K = (S, S_0, R, L)$  and a temporal formula  $\varphi$ , model checking aims to identify  $S' \subseteq S$  such that

$$K, s_i \models \varphi$$

for all  $s_i \in S'$ .

Model checking problem for *CTL\**, *CTL* and *LTL* is decidable even though the time complexity of the algorithm is logics dependent: the complexities of the *CTL*, *LTL* and *CTL\** decision procedures are  $O(|\varphi| * (|S| + |R|))$ ,  $O(2^{O(|\varphi|)} * (|S| + |R|))$  and  $O(2^{O(|\varphi|)} * (|S| + |R|))$ , respectively.

### 1.3.1 Fair Model Checking

A *fair Kripke structure* is a Kripke structure  $(S, S_0, R, L)$  added with a set of *fair states*  $F \subseteq S$ . A *fair path* for it is an infinite path that passes through all the fair states infinitely often.

*Fair model checking* only considers fair paths. A *fair state* is a path from which at least one fair path originates.

## 1.4 Symbolic Representation

*Binary Decision Diagrams* (BDDs) and *Ordered Binary Decision Diagrams* (OBDDs) are data structures to represent binary functions [Bryant86].

### 1.4.1 Binary Decision Diagrams

BDDs are *directed graphs* whose nodes can be either **terminal** or **non-terminal**. Terminal nodes are labelled by a *binary value* and they are not source of any edge. If  $t$  is a terminal node, we write  $t.value$  to denote the value of  $t$ . Non-terminal nodes are labelled by a *variable* name and they are source of two edges called *low* and *high*. If  $n$  is a non-terminal node, we write  $n.var$ ,  $n.low$ , and  $n.high$  to denote the variable name, the edge low, and the edge high of the node  $n$ .

Any terminal node  $t$  represents the binary function  $t.value$ , while any non-terminal node  $n$  encodes the binary function  $(n.var \& f_l) | (n.var \& f_h)$  where  $f_l$  and  $f_h$  are the binary functions associated to  $n.low$  and  $n.high$ , respectively.

A BDD **respects a variable ordering**  $<$  whenever  $n.var < n.low.var$  for all non-terminal nodes  $n$  and  $n.low$  and  $n.var < n.high.var$  for all non-terminal nodes  $n$  and  $n.high$ .

### 1.4.2 Ordered Binary Decision Diagrams

The logical equivalence of two binary functions can be reduced to the existence of an isomorphism between the BDD encoding them under three conditions:

1. the two BDDs respect the same variable ordering;
2.  $n.low$  and  $n.high$  are different nodes for any non-terminal node  $n$  in both the BDDs;
3. for each of the BDDs and for all pairs of nodes in it, there is no isomorphism between them.

OBDDs are BDDs equipped of a variable ordering and satisfying condition 2. and 3.

Whenever two binary functions  $f_1$  and  $f_2$  are stored as OBDD and they share the same variable ordering, it is possible to:

- test logical equivalence between  $f_1$  and  $f_2$  in time  $O(1)$ ;
- compute the OBDD that represents:
  - the bitwise negation of the formula  $f_1$  in time  $O(|f_1|)$ ;
  - the bitwise binary combinations of the functions  $f_1$  and  $f_2$  in time  $O(|f_1| + |f_2|)$ .

## 2.1 Modelling Reactive Systems

### 2.1.1 Directed Graphs

*Directed graphs* can be represented in *pyModelChecking* by using the class `DiGraph` (see *Graph API*).

```
>>> from pyModelChecking import *
>>> G = DiGraph(V=['a',3],
...             E=[('a','a'), ('a','b')])
>>> print(G)

(V=['a', 3, 'b'], E=[('a', 'a'), ('a', 'b')])

>>> G.nodes()

['a', 3, 'b']

>>> G.edges()

[('a', 'a'), ('a', 'b')]

>>> G.add_edge('c','b')
>>> G.nodes()

['a', 'c', 3, 'b']

>>> G.edges()

[('a', 'a'), ('a', 'b'), ('c', 'b')]
```

The same class provides methods to compute **reachable sets**, **reversed graphs** and **subgraphs** of a given directed graph.

```
>>> print(G.get_reversed_graph())

(V=['a', 'c', 3, 'b'], E=[('a', 'a'), ('b', 'a'), ('b', 'c')])

>>> print(G.get_subgraph(['a', 'b', 3]))

(V=['a', 3, 'b'], E=[('a', 'a'), ('a', 'b')])

>>> print(G.get_reachable_set_from(['a', 3]))

set(['a', 3, 'b'])

>>> print(G.get_reachable_set_from(['d']))
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "pyModelChecking/graph.py", line 203, in get_reachable_set_from
    for d in self.next(s):
  File "pyModelChecking/graph.py", line 120, in next
    'of this DiGraph')
RuntimeError: src = 'd' is not a node of this DiGraph
```

*pyModelChecking* can also compute the strongly connected components of a directed graph.

```
>>> G.add_edge('b', 'a')
>>> print(list(compute_strongly_connected_components(G)))
[['a', 'b'], ['c'], [3]]
```

Refer to *Graph API* for more details.

## 2.1.2 Kripke Structures

*Kripke structures* are representable by using the class *Kripke* (see *Kripke API*).

```
>>> from pyModelChecking import *
>>> K = Kripke(S=[0, 1, 3],
...           R=[(0, 2), (2, 2), (0, 1), (1, 0), (3, 2)],
...           L={1: ['p', 'q'], 2: ['p', 'q'], 3: ['q']})
>>> print(K)

(S=[0, 1, 2, 3], S0=set([0]), R=[(0, 1), (0, 2), (1, 0), (2, 2), (3, 2)], L={0: set([0]), 1: set(['q', 'p']), 2: set(['q', 'p']), 3: set(['q'])})
```

The sets of Kripke's states and transitions can be obtained by using the following syntax:

```
>>> K.states()

[0, 1, 2, 3]

>>> K.transitions()

[(0, 1), (0, 2), (1, 0), (2, 2), (3, 2)]
```

It is possible to get the successors of a given state with respect to the Kripke's transitions:

```
>>> K.next(0)

set([1, 2])
```

Finally, the API provides a method for getting the labels of Kripke's states.

```
>>> K.labels()

set(['q', 'p'])

>>> K.labels(3)

set(['q'])
```

## 2.2 Encoding Formulas and Model Checking

*pyModelChecking* provides a user friendly support for building *CTL\**, *CTL* and *LTL* formulas. Each of these languages corresponds to a *pyModelChecking*'s sub-module which implements all the classes required to encode the corresponding formulas.

Propositional logic is also supported by *pyModelChecking* as a shared basis for all the possible temporal logics.

### 2.2.1 Propositional Logics

Propositional logics support is provided by including the *pyModelChecking.language* sub-module. This sub-module allows to represents atomic propositions and Boolean values through the `pyModelChecking.formula.AtomicProposition` and `pyModelChecking.formula.Bool` classes, respectively.

```
>>> from pyModelChecking.formula import *
>>> AtomicProposition('p')

p

>>> Bool(True)

True
```

Moreover, the *pyModelChecking.language* sub-module implements the logic operators  $\wedge$ ,  $\vee$ ,  $\rightarrow$  and  $\neg$  by mean of the classes `pyModelChecking.formula.And`, `pyModelChecking.formula.Or`, `pyModelChecking.formula.Imply` and `pyModelChecking.formula.Not`, respectively. These classes automatically wrap strings and Boolean values as objects of the classes `pyModelChecking.formula.AtomicProposition` and `pyModelChecking.formula.Bool`, respectively.

```
>>> And('p', True)

(p and True)

>>> And('p', True, 'p')

(p and True and p)

>>> f = Implies('q', 'p')
>>> And('p', f, Implies(Not(f), Or('q', 's', f)))
```

(continues on next page)

(continued from previous page)

```
(p and (q --> p) and (not (q --> p) --> (q or s or (q --> p))))

>> Imply('p', 'q', 'p')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: __init__() takes exactly 3 arguments (4 given)
```

For user convenience, the function `pyModelChecking.formula.LNot()` is also provided. This function returns a formula equivalent to logic negation of the parameter and minimise the number of outermost  $\neg$ .

```
>>> f = Not(Not(Not(And('p', Not('q')))))
>>> f

not not not (p and not q)

>>> LNot(f)

(p and not q)

>>> LNot(Not(f))

not (p and not q)

>>> LNot(LNot(f))

not (p and not q)
```

## 2.2.2 Temporal Logics Implementation

CTL\* formulas can be defined by using the `pyModelChecking.CTLS` sub-module.

```
>>> from pyModelChecking.CTLS import *
```

Path quantifiers  $A$  and  $E$  as well as temporal operators  $X$ ,  $F$ ,  $G$ ,  $U$  and  $R$  are provided as classes (see ref: *CTLS sub-module<ctls\_api>* for more details). As in the case of propositional logics, these classes wrap strings and Boolean values as objects of the classes `pyModelChecking.CTLS.language.AtomicProposition` and `pyModelChecking.CTLS.language.Bool`, respectively.

```
>>> phi = A(G(
...     Imply(And(Not('Close'),
...             'Start'),
...     A(Or(G(Not('Heat')),
...         F(Not('Error')))))
... ))
>>> phi

A(G(((not Close and Start) --> A((G(not Heat) or F(not Error)))))
```

In order to simplify the use of the library, a parsing class `pyModelChecking.CTLS.Parser` has been implemented. Its objects read a formula from a string and, when it is possible, translate it into a corresponding `pyModelChecking.CTLS.Formula` objects.



```
>>> parser = Parser()
>>> psi_str = 'A G ((not Close and Start) --> ' +
...           'A(G(not Heat) or F(not Error)))'
>>> psi = parser(psi_str)
>>> psi

A(G(((not Close and Start) --> A((G(not Heat) or F(not Error))))))
```

The sub-module also implements the CTL\* model checking and fair model checking algorithms described in [CGP00].

```
>>> from pyModelChecking import Kripke
>>> K = Kripke(R=[(0, 1), (0, 2), (1, 4), (4, 1), (4, 2), (2, 0),
...              (3, 2), (3, 0), (3, 3), (6, 3), (2, 5), (5, 6)],
...           L={0: set(), 1: set(['Start', 'Error']), 2: set(['Close']),
...              3: set(['Close', 'Heat']),
...              4: set(['Start', 'Close', 'Error']),
...              5: set(['Start', 'Close']),
...              6: set(['Start', 'Close', 'Heat'])})
>>> modelcheck(K, psi)

set([0, 1, 2, 3, 4, 5, 6])

>>> modelcheck(K, psi, F=[6])

set([])
```

It is also possible to model check a string representation of a CTL\* formula by either passing an object of the class `pyModelChecking.CTLS.Parser` or leaving the remit of creating such an object to the function `pyModelChecking.CTLS.modelcheck()`.

```
>>> modelcheck(K, psi_str)

set([0, 1, 2, 3, 4, 5, 6])

>>> modelcheck(K, psi_str, parser=parser)

set([0, 1, 2, 3, 4, 5, 6])
```

Analogous functionality are provided for *CTL* and *LTL* by the sub-modules `pyModelChecking.CTL` and `pyModelChecking.LTL`, respectively.



## 3.1 Models API

*pyModelChecking* provides implementations for *directed graph* and *Kripke structures*.

### 3.1.1 Graph API

It is used to define *directed graphs* and provides a method to compute the strongly connected components of a directed graph.

### 3.1.2 Kripke API

It is used to define *Kripke structures*.

## 3.2 Logics and Model Checking API

The implementations of specific languages and their model checking routines are contained in *pyModelChecking* sub-modules. CTL\*, CTL, and LTL are handled by *CTLS sub-module*, *CTL sub-module* and *LTL sub-module*, respectively.

### 3.2.1 CTLS sub-module API

It represents *CTL\* formulas* and provides model checking methods for them.

#### Language

## Model Checking

### 3.2.2 CTL sub-module API

It represents *CTL formulas* and provides model checking methods for them.

## Language

## Model Checking

### 3.2.3 LTL sub-module API

It represents *LTL formulas* and provides model checking methods for them.

## Language

## Model Checking

## CHAPTER 4

---

### Indices and tables

---

- `genindex`
- `modindex`
- `search`



---

## Bibliography

---

- [P77] A. Pnueli. “The temporal logic of programs.” In Proceedings of the 18th Annual Symposium of Foundations of Computer Science (FOCS), 1977, 46-57
- [BMP83] M. Ben-Ari, Z. Manna, A. Pnueli. The temporal logic of branching time. *Acta Informatica* 20(1983): 207-226
- [CE81] E. M. Clarke, E. A. Emerson. “Design and synthesis of synchronization skeletons using branching time temporal logic.” In *Logic of Programs: Workshop*. LNCS 131. Springer, 1981.
- [CE80] E. M. Clarke, E. A. Emerson. “Characterizing correctness properties of parallel programs using fix-points.” In *Automata, Languages, and Programming*. LNCS 85:169-181. Springer 1980.
- [CES86] E. M. Clarke, E. A. Emerson, A. P. Sistla. “Automatic verification of finite-state concurrent systems using temporal logic specifications.” *ACM Transactions on Programming Languages and Systems* 8(2): 244-263. 1986.
- [CGP00] E. M. Clarke, O. Grumberg, D. A. Peled. “Model Checking” MIT Press, Cambridge, MA, USA. 2000.
- [Bryant86] Randal E. Bryant. “Graph-Based Algorithms for Boolean Function Manipulation”. *IEEE Transactions on Computers*, C-35(8):677–691, 1986.